Topics

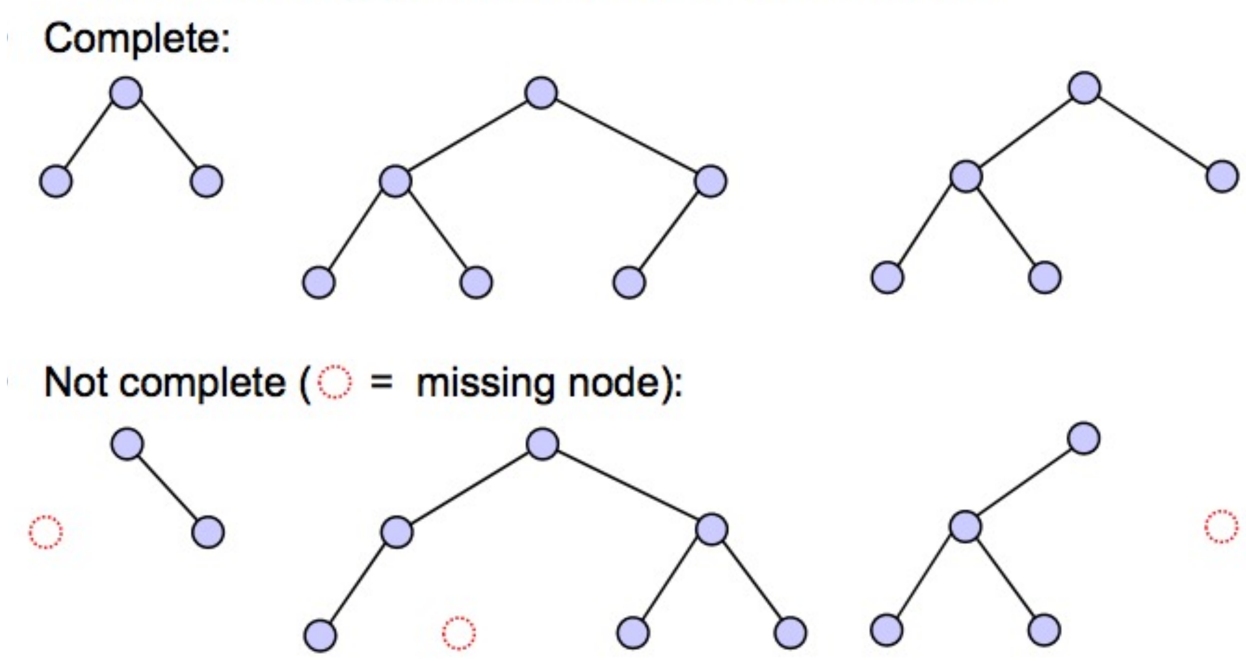
- Heap
- Heapify

Heap

- Not what it sounds like ☺
  - i.e. not a disordered pile of items
- Partially ordered data structure
- Specifically, a heap is a complete binary tree where:
  - The element value of each parent node is greater than or equal to the element values its children (**max heap**)
  - Or, the element value of each parent node is less than or equal to the element values its children (*min heap*)

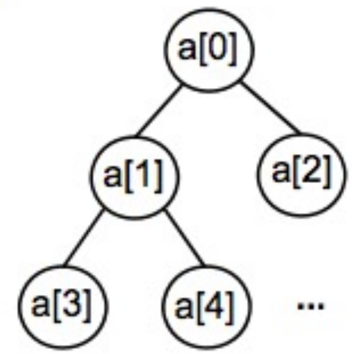
Complete Binary Tree?

- A binary tree of height ***h*** is complete if:
  - Levels ***0*** through ***h-1*** are fully occupied
  - There are no gaps to the left of a node in level ***h***

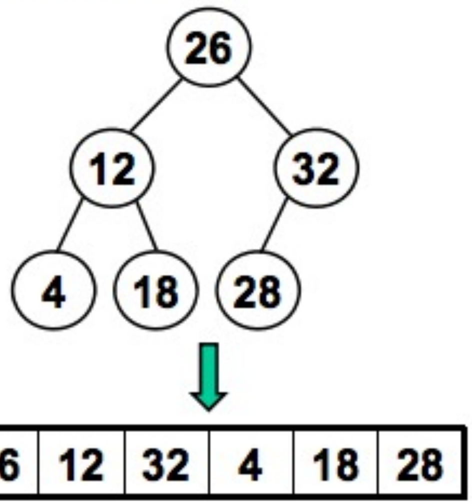


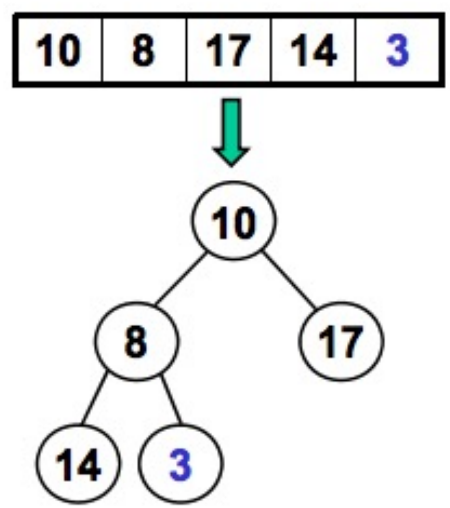
Why complete binary tree?

- Has simple array representation
- Nodes are stored in the order visited
  - Top to bottom
  - Left to right



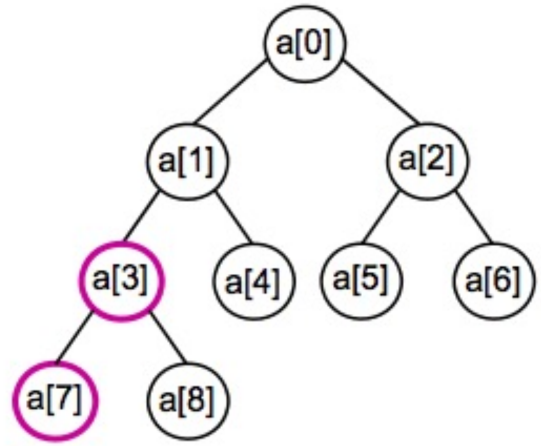
Examples:





Nodes and Index Positions

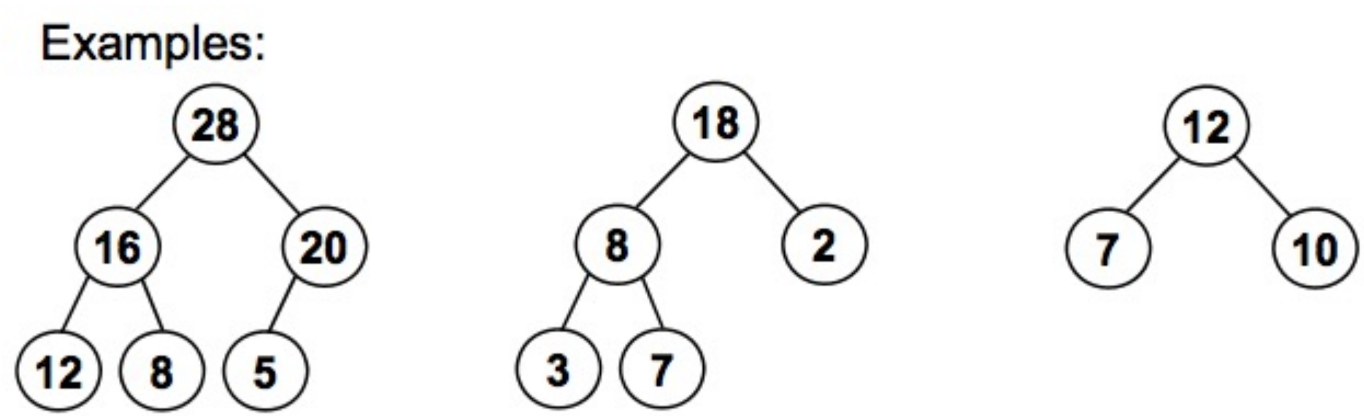
- Root node is `A[0]`
- Given node at `A[i]`
  - Left child `A[ 2*i + 1 ]`
  - Right child `A[ 2*i + 2 ]`
  - Parent `A[ floor( ( i - 1 ) / 2 ) ]`



- Examples
  - Left child of `A[1]: A[2*1+1] = A[3]`
  - Right child of `A[3]: A[2*3+2] = A[8]`
  - Parent of `A[4]: A[floor((4-1)/2)] = A[1]`

Max Heap Examples

- Largest value is always root node of tree
- Smallest value can be any leaf node
  - No guarantee which one it will be …

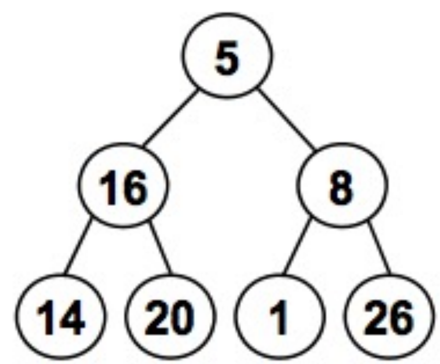


Max Heapify

- Convert an ordinary list of items to a heap
- Bottom up approach

Example:

0	1	2	3	4	5	6
5	16	8	14	20	1	26

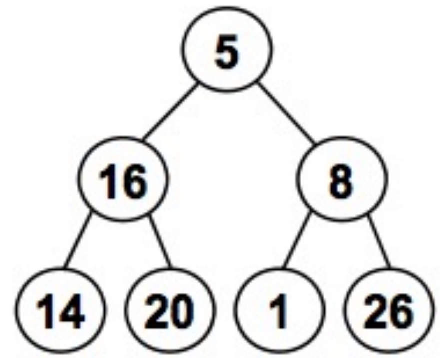


Max Heapify Pseudocode

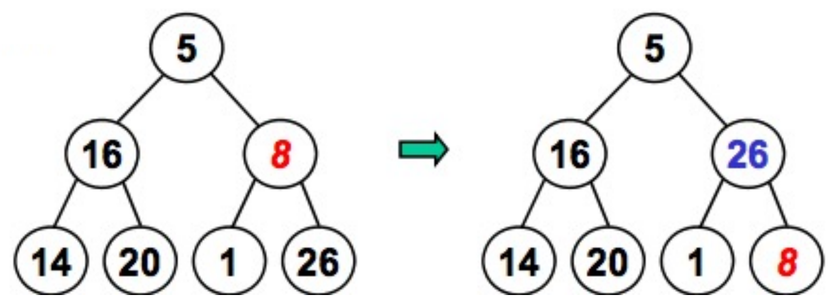
```
ALGORITHM maxHeapify( H[0 … n-1] )
// Constructs a heap from an existing list of values
// Input: list H
// Output: heap H
for i = floor( ( n-2 )/2 ) downto 0 do
        k = i, v = H[k]
        heap = false
        while not heap and 2*k+2 <= n do
                j = 2*k+1
                if j+1 < n // two children
                        if H[j] < H[j+1], j = j + 1
                if v >= H[j]
                        heap = true
                else
                        H[k] = H[j] // swap parent and largest child
                        k = j
```

Example

0	1	2	3	4	5	6
5	16	8	14	20	1	26



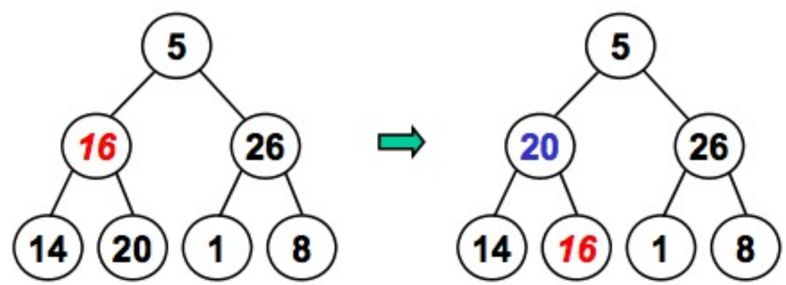

```
k = 2, v = 8, heap=false

      while 2*k+2 <= 7 and not heap

      j = 5 (5+1 < 7 -> two children)

      j = 6 ( 26 > 1 )

      H[2] < H[6] -> H[2] = H[6]

      k = 6
```



Example

0	1	2	3	4	5	6
5	16	26	14	20	1	8

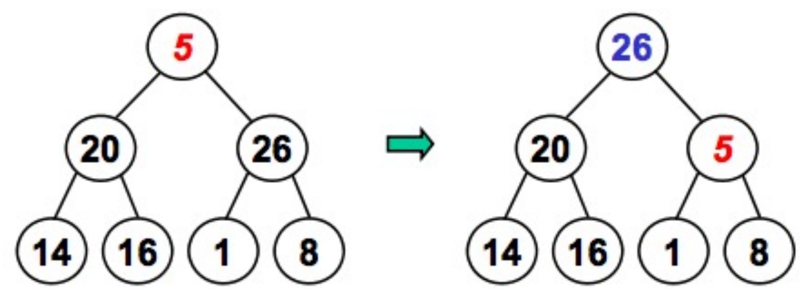
```
k = 1, v = 16, heap=false

        while 2*k+2 <= 7 and not heap

        j = 3 (3+1 < 7 -> two children)

        j = 4 ( 20 > 14 )

        H[1] < H[4] -> H[1] = H[4]

        k = 4
```



Example

0	1	2	3	4	5	6
5	20	26	14	16	1	8

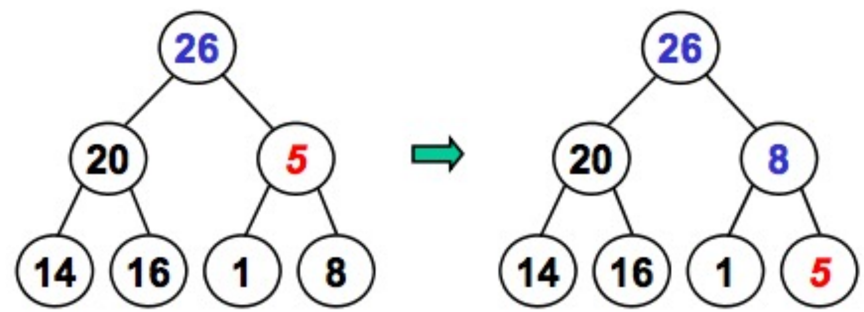
```
k = 0, v = 5, heap=false

        while 2*k+2 <= 7 and not heap

        j = 1 (1+1 < 7 -> two children)

        j = 2 ( 26 > 20 )

        H[0] < H[2] -> H[0] = H[2]

        k = 2
```



Example

0	1	2	3	4	5	6
26	20	5	14	16	1	8

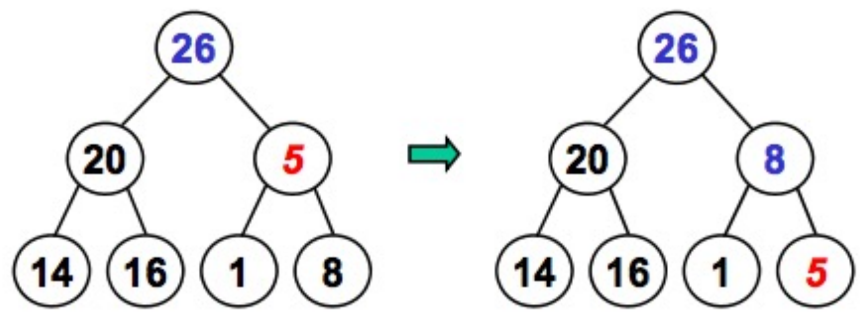
```
k = 0, v = 5, heap=false

        while 2*k+2 <= 7 and not heap

        j = 5 (5+1 < 7 -> two children)

        j = 6 ( 8 > 1 )

        H[2] < H[6] -> H[2] = H[6]

        k = 6
```



Example

0	1	2	3	4	5	6
26	20	8	14	16	1	5

Done



Min Heap?

- Opposite of Max Heap
  - Smallest value is always root node of tree
  - Largest value can be any leaf node
    - No guarantee which one it will be …