Discrete Math: Homework 8

Due on November 16, 2023 at 11:59pm
Tuesday/Thursday 11:00-12:15, Phillips 383

Reese Lance - Section 003

Rushil Umaretiya
rumareti@unc.edu

Unit 4.1

#10 Prove that if a and b are nonzero integers, $a|b$, and $a+b$ is odd, then a is odd.

Proof. Direct proof. Assume a and b are nonzero integers, $a|b$, and $a+b$ is odd. Then, by definition of divisibility, $\exists k \in \mathbb{Z}, ak = b$. We also know that $a+b$ is odd, so $\exists j \in \mathbb{Z}, 2j+1 = a+b$. Substituting b for ak, we get $2j+1 = a+ak$. Factoring out a, we get $2j+1 = a(1+k)$. Since $1+k$ is an integer, we know that a is odd. □

#26 Evaluate these quantites:

a) $-17 \ \mathbf{mod} \ 2 =$ 1

b) $144 \ \mathbf{mod} \ 7 =$ 4

c) $-101 \ \mathbf{mod} \ 13 =$ 3

d) $199 \ \mathbf{mod} \ 19 =$ 9

#34 Decide whether each of these integers is congruent to 3 modulo 7.

a) 37

$$\begin{aligned} 7 &\nmid (37 - 3) \\ 37 &\not\equiv 3 \pmod{7} \end{aligned}$$

b) 66

$$\begin{aligned} 7 &\mid (66 - 3) \\ 66 &\equiv 3 \pmod{7} \end{aligned}$$

c) -17

$$\begin{aligned} 7 &\nmid (-17 - 3) \\ -17 &\not\equiv 3 \pmod{7} \end{aligned}$$

d) -67

$$\begin{aligned} 7 &\mid (-67 - 3) \\ -67 &\equiv 3 \pmod{7} \end{aligned}$$

#36 Find each of these values.

Note: The following rules exist for modular arithmetic where $a, b, n \in \mathbb{Z}, n \geq 2$:

$$(a + b) \textbf{ mod } n = ((a \textbf{ mod } n) + (b \textbf{ mod } n)) \textbf{ mod } n \qquad \text{Addition Law}$$
$$(a \cdot b) \textbf{ mod } n = ((a \textbf{ mod } n) \cdot (b \textbf{ mod } n)) \textbf{ mod } n \qquad \text{Multiplication Law}$$

a) $(177 \textbf{ mod } 31 + 270 \textbf{ mod } 31) \textbf{ mod } 31$

$$\begin{aligned}(177 \textbf{ mod } 31 + 270 \textbf{ mod } 31) \textbf{ mod } 31 &= (177 + 270) \textbf{ mod } 31 \\ &= 447 \textbf{ mod } 31 \\ &= 13\end{aligned}$$

b) $(177 \textbf{ mod } 31 \cdot 270 \textbf{ mod } 31) \textbf{ mod } 31$

$$\begin{aligned}(177 \textbf{ mod } 31 \cdot 270 \textbf{ mod } 31) \textbf{ mod } 31 &= (177 \cdot 270) \textbf{ mod } 31 \\ &= 47790 \textbf{ mod } 31 \\ &= 19\end{aligned}$$

Unit 6.1

#8 How many different three-letter initials with none of the letters repeated can people have?

$$26 \cdot 25 \cdot 24 = 15600$$

#22 How many positive integers less than 1000,

a) are divisible by 7?

$$\lfloor \frac{1000}{7} \rfloor = 142$$

b) are divisible by 7 but not by 11?

$$\lfloor \frac{1000}{7} \rfloor - \lfloor \frac{1000}{77} \rfloor = 142 - 12 = 130$$

c) are divisible by both 7 and 11?

$$\lfloor \frac{1000}{77} \rfloor = 12$$

d) are divisible by either 7 or 11?

$$\lfloor \frac{1000}{7} \rfloor + \lfloor \frac{1000}{11} \rfloor - \lfloor \frac{1000}{77} \rfloor = 142 + 90 - 12 = 220$$

e) are divisible by exactly one of 7 and 11?

$$\lfloor \frac{1000}{7} \rfloor + \lfloor \frac{1000}{11} \rfloor - 2\lfloor \frac{1000}{77} \rfloor = 142 + 90 - 24 = 208$$

f) are divisible by neither 7 nor 11?

$$999 - 220 = 779 \tag{1}$$

g) have distinct digits?

$$\begin{aligned} 9 & \quad \text{One digit} \\ (10-1)\cdot 9 & \quad \text{Two digits} \\ (10-1)\cdot 9 \cdot 8 & \quad \text{Three digits} \\ 9 + 9\cdot 9 + 9\cdot 8\cdot 7 = 738 & \quad \text{Total} \end{aligned}$$

h) have distinct digits and are even

$$\begin{aligned} 4 & \quad \text{One digit} \\ 9 + 8\cdot 4 & \quad \text{Two digits} \\ 9\cdot 8 + 8\cdot 8\cdot 4 & \quad \text{Three digits} \\ 4 + (9 + 8\cdot 4) + (9\cdot 8 + 8\cdot 8\cdot 4) = 373 & \quad \text{Total} \end{aligned}$$

#28 How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

$$10^3 \cdot 26^3 + 26^3 \cdot 10^3 = 3.5152 \cdot 10^7$$